Quiz 3, MATH 240, Fall 2023

Write your name clearly.

Name:

Section Number: UID:

(1) Let A be a matrix that has the RREF
$$\begin{pmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(a) [8] Write the parametric vector form of the solution set to $A\mathbf{x} = \mathbf{0}$.

(b) [8] If we know $A \begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, write the parametric vector form of the solution

set to $A\mathbf{x} = \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}$.

(c) [4] Are the columns of A linearly independent or linearly dependent? Explain.

(a) The solutions to $A\vec{x} = \vec{0}$ are given by

$$x_1 = 2x_3 - 3x_4$$
$$x_2 = x_4$$
$$x_3, x_4 \text{ free},$$

i.e. $\quad \vec{x} = x_3 \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -3 \\ 1 \\ 0 \\ 1 \end{pmatrix}$.

(b) Note that $A(-2, 1, 0, -1) = (-1, -1, -1, -1)$. So the solutions to $A\vec{x} = (-1, -1, -1, -1)$ are given by

$$\vec{x} = (-2, 1, 0, -1) + x_3(2, 0, 1, 0) + x_4(-3, 1, 0, 1).$$

(c) The columns of A are linearly dependent because its RREF does not have a pivot in every column.